Examinee No.	

(Fill in your examinee number above)

2023 Academic Year

The University of Tokyo, Graduate School of Engineering

Entrance Examination

for

Department of Aeronautics and Astronautics

Afternoon Session

(13:30～16:30)

IMPORTANT NOTICES

1. Do not open this booklet before the start of the examination.
2. Choose and answer the questions from three fields out of four.
3. Three answer sheets are given. Use each sheet to answer the problems of different field.
4. Fill the field name and your examinee number in the answer sheet.
5. Do not carry out this booklet nor the answer sheets.

Fluid Dynamics (Afternoon)

Consider a compressible inviscid quasi-one-dimensional steady flow of an ideal gas in the direction of x-axis. The specific heat ratio of the gas is γ. The velocity u , Mach number M , pressure p , density ρ , and the cross-sectional area of the stream tube A are the functions of x. Answer the following questions. Assume the isentropic flow in Questions 1-3.

Question 1

Derive the equation (1) from the conservation of the total enthalpy. p_0 is the stagnation pressure.

$$\frac{p}{p_0} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\gamma/(\gamma-1)} \tag{1}$$

Question 2

Using the fact that the mass flow rate in the stream tube $\rho u A$ is constant, and the equation of motion

$$\rho u \frac{du}{dx} = -\frac{dp}{dx} , \tag{2}$$

derive the equation (3).

$$\frac{1}{A}\frac{dA}{dx} = \frac{1 - M^2}{\gamma M^2 p}\frac{dp}{dx} \tag{3}$$

Question 3

Derive the equation (4).

$$\frac{1}{A}\frac{dA}{dx} = -\frac{1 - M^2}{M^2\{2 + (\gamma - 1)M^2\}}\frac{dM^2}{dx} \tag{4}$$

Question 4

Explain qualitatively using a figure how the distribution of the Mach number in a de Laval nozzle changes, depending on the back pressure p_b.

Question 5

When p_b is in a certain range, the normal shock wave appears in the de Laval nozzle. The equation (5) holds

$$\frac{p_2}{p_1} = \frac{\frac{\gamma + 1}{\gamma - 1}\frac{\rho_2}{\rho_1} - 1}{\frac{\gamma + 1}{\gamma - 1} - \frac{\rho_2}{\rho_1}} , \tag{5}$$

where the subscripts 1 and 2 denote the conditions just before and just after the normal shock wave, respectively. Prove that the pressure rise by the normal shock wave is larger than that by the isentropic compression with the same density change.

Solid Mechanics (Afternoon)

A composite plate composed by material 1 and material 2 supports the tensile load P. Answer the following questions. Denote the thickness of the plate as t, the width as w, the length as L, where L is much larger than t and w. Material 1 and material 2 are isotropic. Their Poisson's ratios ν are identical. Bonding between material 1 and material 2 is perfect. Young's modulus E_1 of material 1 is larger than E_2 of material 2 ($E_1 > E_2$) and tensile strength σ_2^B of material 2 is larger than σ_1^B of material 1 ($\sigma_1^B < \sigma_2^B$). Both materials deform elastically until fracture. Assume that broken plates support no load after fracture. You can define and use the necessary terms and symbols, which are not given here.

Question 1 Consider a uni-axial tensile test of the composite plate as shown in Figure 1. Derive averaged Young's modulus of the composite plate. You must show the derivation process in the answer.

Question 2 Illustrate and describe the averaged stress and strain relation of the composite plate as shown in Figure 1 until final fracture under a uni-axial tensile test with constant displacement speed.

Question 3 Consider a uni-axial tensile test of the composite plate as shown in Figure 2. Derive averaged Young's modulus of the composite plate. You must show the derivation process in the answer.

Question 4 Illustrate and describe the averaged stress and strain relation of the composite plate as shown in Figure 2 until final fracture under a uni-axial tensile test with constant displacement speed.

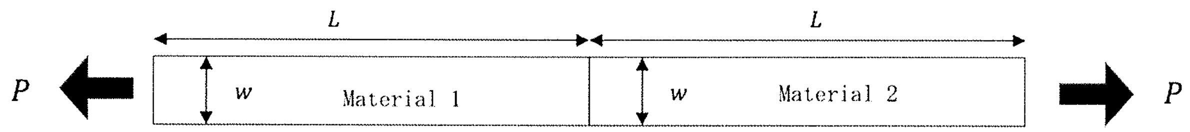

Figure 1

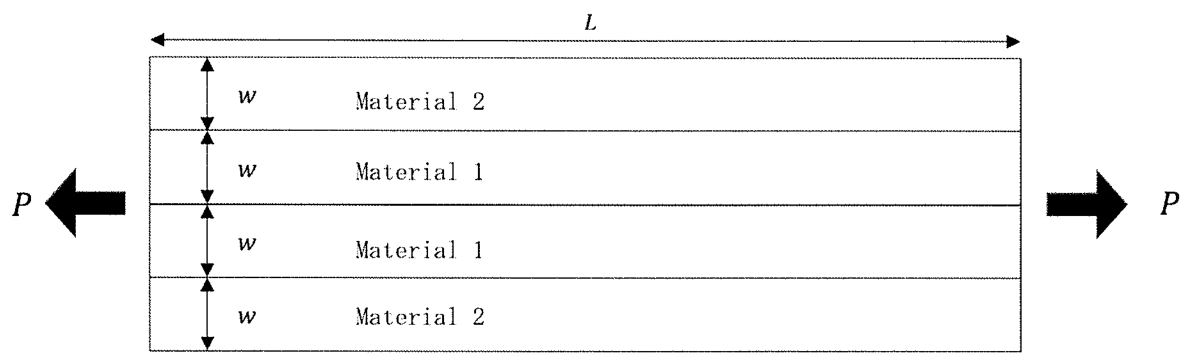

Figure 2

Aerospace System (Afternoon)

In order to remove debris, we want to reduce the height of satellites or rockets which have ended their life time but still stay in orbits, with radiation pressure exerted by emitting laser light to them from ground. Answer the following questions. If the questions cannot be solved without setting some assumptions, write the assumptions you made as well. If you want to use additional variables or constants in expressions or discussions, define them before using them.

Question 1

Explain qualitatively using drawings and expressions how satellite attitude and orbit are generally changed if the satellite gets radiation pressure of laser light or sun light.

Question 2

The Earth is assumed to be a sphere of radius r_0. As shown in Figure 1, laser light is emitted from laser station A on the ground to satellite B flying on a circular orbit of radius r. Laser station A is located on the orbital plane of the orbit of satellite B. The angle between the direction of the emitted laser light and the local vertical at the position of laser station A is θ, and laser light tracks the satellite B by changing θ. Assuming that this laser light gives satellite B a force F along the direction of the laser light, derive the elements of this force along the velocity vector direction and along the radial direction of the orbit of satellite B, as functions of θ. Solar radiation pressure, atmospheric drag, effects of the self-rotation of the Earth, and the attenuation of laser light by atmosphere can be neglected.

Question 3

Derive the expression of time derivative of the orbital angular momentum of satellite B caused by force F in Question 2.

Question 4

This laser light can be turned on or off, while tracking satellite B. We want to reduce the orbital angular momentum of satellite B efficiently by reducing the time when the laser light is turned on as much as possible. Discuss qualitatively, using expressions and figures if necessary, at what timing laser light should be turned on, for the following two cases. The direction of force F is the same as that defined in Question 2.

1. The magnitude of force F is constant.
2. The magnitude of force F is inversely proportional to the square of the distance between laser station A and satellite B.

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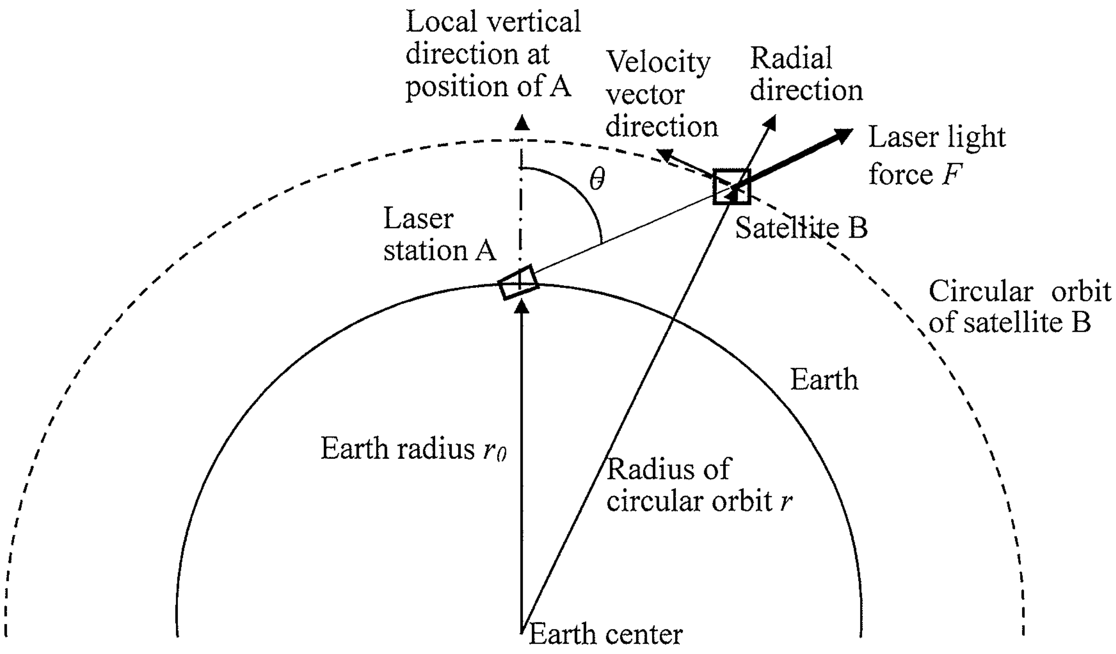
Local vertical direction at position of A
Velocity vector direction
Radial direction
Laser light force F
θ
Laser station A
Satellite B
Circular orbit of satellite B
Earth
Earth radius r_0
Radius of circular orbit r
Earth center

Figure 1

Propulsion Engineering (Afternoon)

Question 1

As shown in Figure 1, a shaft with an unbalanced disk is rotating around the center of the bearing at a constant angular velocity ω. The disk is modeled as a concentrated mass m at G. Ignore the thickness of the disk and the thickness of the shaft. Figure 2 shows the disk viewed in the shaft direction. The center of the shaft C at the disk position is rotating around the center of the bearing O in the stationary condition with a displacement r. The center of gravity G of the disk has an eccentricity ε from the shaft center C, and rotates around C at an angular velocity ω_g. In the stable whirling, the angular velocity of the rotation ω and the angular velocity of the center of gravity around the shaft center ω_g match each other, and the disk whirls in the manner shown in Figure 2. Let k be the equivalent spring constant of the shaft, and the damping and the torsion are ignored.

1. In the case without the unbalance in which G matches to C, find the whirling frequency of the shaft from the balance between the centrifugal force and the restoring force of the shaft.

2. Consider the case with the unbalance hereafter. In the coordinate system of Figure 2, the coordinate of the shaft center z is expressed as $z = x + iy$ (i : imaginary unit). Express the coordinate of the center of gravity G, z_g, using z, ε, ω_g, and time t.

3. Derive the equation which describes the whirling oscillation of the disk with an unbalance mass in the stable whirling state.

4. Obtain the amplitude of the forced response solution of the equation in the previous question, and find the condition under which the oscillation diverges. Use the natural frequency of the system ω_n corresponding to the case without unbalance.

5. Explain what kind of whirling motion occurs depending on the magnitude relationship between ω_g and ω_n. Attach a figure showing the positional relationship between O, C, and G for each case.

6. What kind of whirl occurs in the limit case of increasing angular velocity of the rotation?

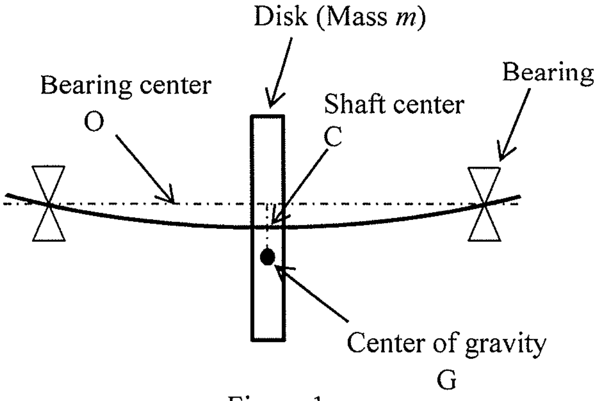

Figure 1

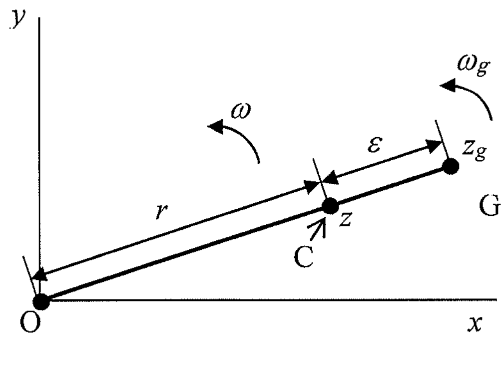

Figure 2

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Question 2

A motor with a flywheel is supported on the floor by a spring and a dashpot as shown in Figure 3. The motor rotates at an angular velocity ω. The spring constant is k and the viscous damping coefficient of the dashpot is c. The x axis is taken in the vertically downward direction from the equilibrium position. The entire motor system including the flywheel moves only in the x direction. An unbalance mass m is attached to the flywheel with a distance ε from the shaft center of the motor. Let M be the total mass of the motor and the unbalance mass.

1. Derive the equation of motion of the motor.
2. From the equation of motion, obtain the displacement of the motor after the steady oscillation is attained.
3. Obtain the displacement amplitude of the steady oscillation.
4. What happens to the amplitude when the system resonates?
5. What happens to the amplitude at the limit case of increasing angular velocity of the motor rotation?
6. In the limit situation above, show the positional relationship between the shaft center, the center of mass, and the unbalance mass using a diagram along with the reason. Describe the characteristics of the motion in this situation.

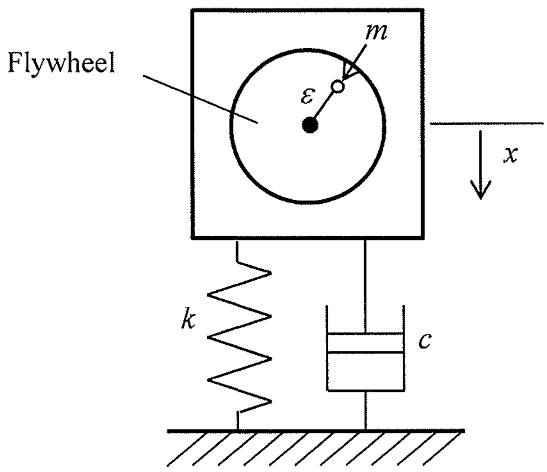

Figure 3