Examinee No.	

(Fill in your examinee number above)

2023 Academic Year

The University of Tokyo, Graduate School of Engineering

Entrance Examination

for

Department of Aeronautics and Astronautics

Morning Session

(9:00～12:00)

IMPORTANT NOTICES

1. Do not open this booklet before the start of the examination.
2. Choose and answer the questions from three fields out of four.
3. Three answer sheets are given. Use each sheet to answer the problems of different field.
4. Fill the field name and your examinee number in the answer sheet.
5. Do not carry out this booklet nor the answer sheets.

Fluid Dynamics (Morning)

There is liquid, whose density is ρ, in a vessel placed in a uniform pressure p_∞, as shown in Figure 1. A uniform diameter slender pipe, whose length is L, is connected to the vessel horizontally. The height of the liquid surface from the central axis of the pipe is h. Let the inlet of the pipe be "1," and the end of the pipe be "2." The cock at the end of the pipe "2" is suddenly opened at the time $t = 0$. Let the velocity of the liquid efflux from "2" be q. The liquid is assumed inviscid and incompressible, and its flow is assumed irrotational. The vessel volume is sufficiently large, and the change in the liquid surface height associated with the liquid efflux is assumed negligible. Let the magnitude of the gravitational acceleration be g.

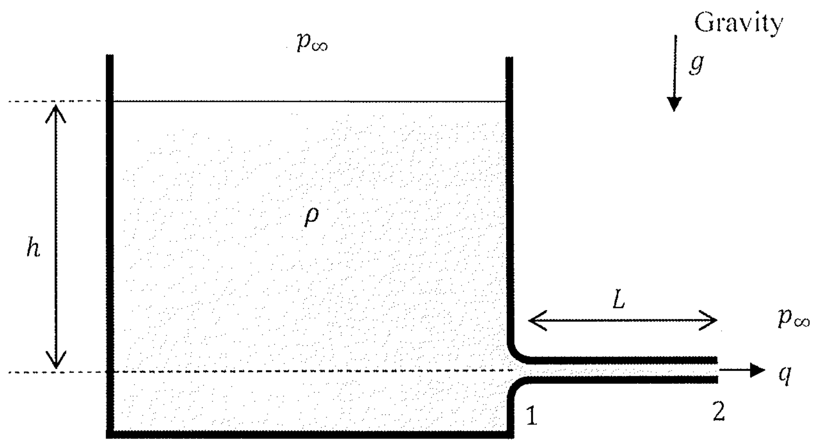

Figure 1

Question 1

If the flow is considered steady after a sufficiently long time has passed since the cock was opened, obtain q_∞ which is the velocity of liquid efflux at the end of the pipe "2."

Question 2

The Euler's equation of fluid is given by Eq. (1).

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$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \mathrm{grad})\vec{v} = \vec{g} - \frac{\mathrm{grad}\ p}{\rho} \tag{1}$$

where, $\vec{v}$ is the velocity vector of the fluid, p is the pressure of the fluid, $\vec{g}$ is the gravitational acceleration vector.

Let ϕ be the velocity potential. Derive Eq. (2) from Eq. (1). You may use the relation $(\vec{A} \cdot \mathrm{grad})\vec{A} = \mathrm{grad}\left(\frac{|\vec{A}|^2}{2}\right) - \vec{A} \times \mathrm{rot}\,\vec{A}$ that holds for arbitrary vector $\vec{A}$.

$$\mathrm{grad}\left(\frac{\partial \phi}{\partial t} + \frac{1}{2}|\vec{v}|^2 + \frac{p}{\rho}\right) = \vec{g} \tag{2}$$

Question 3

Express the velocity of liquid efflux q at the end of the pipe "2" in terms of t. Show that $q \to q_\infty$ when $t \to \infty$.

Question 4

Express the pressure at the inlet of the pipe p_1 in terms of t.

Solid Mechanics (Morning)

Question 1

Figure 1 represents a cantilevered honeycomb sandwich beam of width b and length L. The thickness and the Young's modulus of two face sheets are t and E. The height and the equivalent modulus of elasticity in shear of the core made of honeycomb are h and G_C. And the distributed load and the bending displacement are $p(x)$ and $w(x)$. For the purpose of simplicity, the followings are assumed.

- The core does not transmit the axial load in the x direction.
- $(h - t)$ is able to be approximately expressed as h, because $h \gg t$.
- The bending rigidity of each face sheet is not considered.
- The bending displacement $w(x)$ in the face sheets and the core is identical regardless of z.

Figures 2 and 3 show the equilibrium of the loads and the deformation in an infinitesimal element of length dx shown in Figure 1, respectively. The axial load and the displacement of the face sheets in the x direction and the shearing load of the core are $F(x), u(x),$ and $Q(x)$. Answer the following questions.

1. Derive the equilibrium equations of the loads in the z direction and the moments in the core, respectively, using Figure 2.
2. Derive the equation indicating the relation between $F(x)$ and $u(x)$ according to the Hooke's law.
3. Derive the equation indicating the relation between $u(x)$ and $p(x)$.
4. Express the shearing strain γ_{xz} in the xz plane of the core, using Figure 3.
5. Derive the equation indicating the relation between $u(x)$ and $w(x)$.
6. Obtain $w(x)$ in case that the concentrated load P in the $+z$ direction is applied at $x = L$, instead of applying $p(x)$.

Question 2

A regular hexagonal honeycomb core shown in Figure 4 is applied to the sandwich beam in Question 1. The honeycomb core and the face sheets are made of the same material of the Young's modulus E, the density ρ, and the Poisson's ratio ν. The side length and thickness of every honeycomb wall are a and e. The equivalent modulus of elasticity in shear G_C of the honeycomb core is expressed as

$$G_C = \frac{\sqrt{3}eG}{3a} \qquad (1)$$

where G is the modulus of elasticity in shear of a material used for the honeycomb wall.

The core height h is configured so that the bending displacement $w(L)$ derived from the answer of Question 1.6 becomes as small as possible on the condition that the mass per unit length μ of the beam is constant. Explain the increase or decrease of the optimal h as the beam length L increases, and

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derive the asymptotic value of the optimal h. No buckling needs to be considered. Use the following equation (2), if necessary.

$$G = \frac{E}{2(1+\nu)} \tag{2}$$

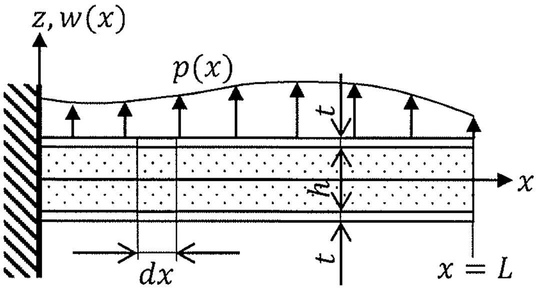

Figure 1

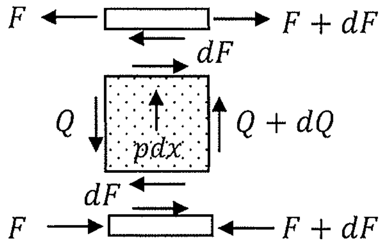

Figure 2

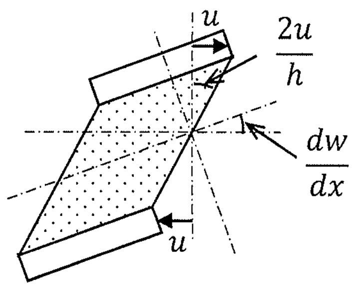

Figure 3

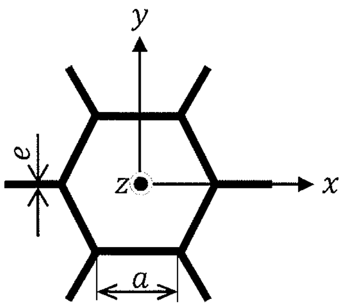

Figure 4

Aerospace System (Morning)

Consider a system in which two carts are connected by springs and dampers shown in Figure 1. k_1 and k_2 are spring constants, c_1 and c_2 are damping coefficients, M_1 and M_2 are masses of Cart 1 and Cart 2, respectively. $x_1(t)$ and $x_2(t)$ are horizontal positions of Cart 1 and Cart 2 and each origin is the position where both springs are equilibrium lengths. When a horizontal force $f(t)$ acts on Cart 2, the carts move. The position of the carts and the force acting on Cart 2 are both positive on the right direction. The carts move smoothly in the horizontal direction on the floor, and the moment of inertia of the wheels are not considered. The wall and floor do not move or deform. Answer the following questions. If any assumptions, variables, or constants are needed to answer, define them before using them.

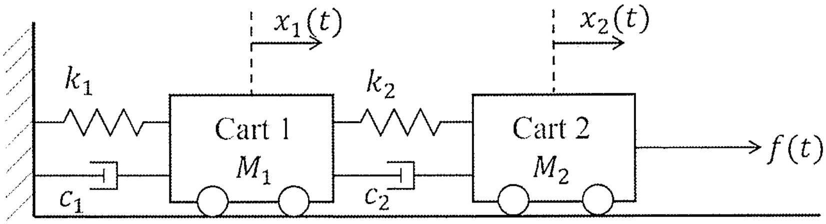

Figure 1

Question 1

We regard $[x_1\ x_2\ \dot{x}_1\ \dot{x}_2]^T$ as the state variable vector, and $f(t)$ as the input variable. $\dot{x}$ is the time derivative of x. Derive the state equation for this system.

Question 2

It is assumed that only the value of $x_1(t)$ can be directly observed using a potentiometer. Derive the output equation for this system, where the output variable is $y(t) = x_1(t)$.

In the remaining, we consider $k_1 = k_2 = c_1 = c_2 = M_1 = M_2 = 1$. The state equation for this system is as follows.

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 1 & -2 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} f$$

Question 3

Evaluate the observability of this system. The state variables, input variable, and output variable are defined in the same way as in Questions 1 and 2.

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Question 4

Laplace transforms of $f(t)$, $x_1(t)$, and $x_2(t)$ are $F(s)$, $X_1(s)$, and $X_2(s)$, respectively. Obtain the transfer function $G(s)$ from $F(s)$ to $X_1(s)$.

Question 5

It is assumed that $f(t)$ is generated by rotating the wheels of Cart 2 with a motor, and the input power to the motor is $w(t)$. Consider a proportional feedback control using the observation variable $x_1(t)$, where the input power to the motor $w(t)$ is used as the control variable. The target value of $x_1(t)$ is given as $r(t)$, and its Laplace transform is $R(s)$. The Laplace transform of $w(t)$ is $W(s)$, and the transfer function from $W(s)$ to $F(s)$ is $H(s)$. The proportional gain of the feedback control is K. Draw a block diagram for this control system using K, $G(s)$, $H(s)$, $R(s)$, $W(s)$, $F(s)$, and $X_1(s)$.

Question 6

Assuming $H(s) = \frac{1}{s+1}$ in the feedback control system defined in Question 5, find the condition of K for this system to be stable.

Propulsion Engineering (Morning)

Technologies to obtain energy from waste heat are attracting attention. Consider the performance of a Stirling engine and a thermoelectric generator using a hot source (temperature T_{H}) and a cold sink (temperature T_{L}), where $T_{\mathrm{H}}/T_{\mathrm{L}} = 1.3$.

Question 1

A Stirling engine is the one in which a gas sealed inside a cylinder is externally heated and cooled to obtain work through changes in its volume (expansion by heating and compression by cooling), and consists of isothermal compression, constant volume heating, isothermal expansion, and constant volume cooling processes. Let the engine compression ratio be σ and assume that the gas sealed in the cylinder is a perfect gas with mass m, gas constant R, and specific heat ratio 1.4. Answer the following questions.

1. Draw a pressure-volume diagram and a temperature-entropy diagram.
2. Find the work done by the gas externally, the change in internal energy, and the energy obtained from the hot source in the process of constant volume heating.
3. Find the work done by the gas externally, the change in internal energy, and the energy obtained from the hot source in the process of isothermal expansion.
4. Calculate the theoretical thermal efficiency. Assume $\ln \sigma = 1.5$.
5. Calculate the theoretical thermal efficiency when the energy lost to the cold sink in the constant volume cooling process can be reversibly used for heating in the constant volume heating process.

Question 2

A thermoelectric generator is a device in which two different electrically conductive materials are placed in contact with one hot source and the other cold sink as shown in Figure 1 to generate an electric potential difference $V = \varepsilon(T_{\mathrm{H}} - T_{\mathrm{L}})$ in the circuit, where the thermoelectric coefficient ε is determined by the combination of the two materials. When a current I flows in the circuit, heat absorption (conversion of heat flux to electrical power) of $Q_{\varepsilon} = \varepsilon I T_{\mathrm{H}}$ occurs in the hot source part. Let the combined thermal conductivity of the two materials be Λ and answer the following questions. Assume no temperature dependence for each property.

1. If the Joule heat rI^2 generated in the conductive materials (combined internal resistance r) is evenly transferred to the hot source and cold sink parts, find the heat flux Q obtained from the hot source.

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2. Calculate the available electric power $r_L I^2$ when a load resistor (resistance r_L) is connected to the circuit. Assume $r = 0.40 \times 10^{-3}$ Ω, resistance ratio $r_L/r = 1.5$, $\varepsilon = 0.40 \times 10^{-3}$ V/K, $T_L = 300$ K and $\Lambda = 0.10$ W/K.

3. Calculate the theoretical efficiency (a fraction of heat flux from the hot source that is converted to the electric power at the load resistor) of the above sub-question 2.

4. Calculate the resistance ratio r_L/r that maximizes the theoretical efficiency when r_L is variable. Square roots need not be calculated.

5. Calculate the theoretical efficiency when the effects of Joule heat generated by the internal resistance and heat conduction in the conductive materials are small and negligible.

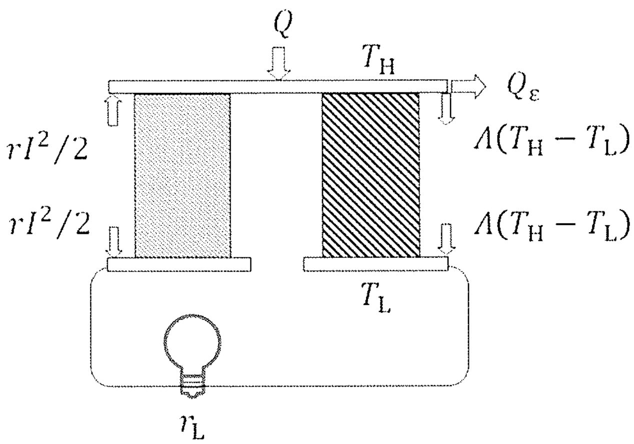

Figure 1 Schematic of a thermoelectric generator circuit.