Examinee No.

(Fill in your examinee number above)

2022 Academic Year

The University of Tokyo, Graduate School of Engineering

Entrance Examination

for

Department of Aeronautics and Astronautics

Morning Session

(9:00~12:00)

IMPORTANT NOTICES

- 1. Do not open this booklet before the start of the examination.
- 2. Choose and answer the questions from three fields out of four.
- 3. Three answer sheets are given. Use each sheet to answer the problems of different field.
- 4. Fill the field name and your examinee number in the answer sheet.
- 5. Do not carry out this booklet nor the answer sheets.

Fluid Dynamics (Morning)

Boundary layer suction is one of the techniques to control the flow around the aircraft. Here we think about the laminar boundary layer formed on semi-infinite two-dimensional flat plate placed at zero incidence against the free stream in an incompressible viscous steady flow. As shown in Figure 1, *x*-*y* coordinate system is defined and velocity components along *x* axis and *y* axis are expressed as *U* and *V*, respectively. Free stream velocity is U_{∞} . There is no pressure gradient along the plate surface. All the plate surface has numerous microscale holes and uniform suction is achieved along whole region of plate surface at a constant velocity of V_0 (>0) with the help of a suction pump placed underneath the plate. Thus, at y=0, U=0 and $V=-V_0$ are satisfied. Use ρ as the air density and v as the coefficient of kinematic viscosity, both of them are constant. In the following, subscript 0 denotes the plate surface (y=0) and subscript f denotes *x* position far downstream from the leading-edge. Answer the following questions.



Question 1 Consider the laminar boundary layer far downstream from the leading-edge of this semiinfinite flat plate.

1. Obtain the velocity distribution of $\left(\frac{U}{U_{\infty}}\right)_{\rm f}$ as a function of y using Equations (1) and (2). Let us

consider the flow characteristics far downstream from the leading-edge are independent of x.

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$
(1)
$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = -\frac{1}{\rho} \frac{dP}{dx} + V \frac{\partial^2 U}{\partial y^2} ,$$
(2)

where *P* is pressure.

2. Obtain the displacement thickness $\delta_{\rm f}^*$ and the viscous shearing stress at the plate surface $(\tau_{\rm f})_0$.

The definition of displacement thickness is $\delta^* \equiv \int_0^\infty (1 - \frac{U}{U_\infty}) dy$.

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Question 2 We think about obtaining an approximate velocity profile $\frac{U}{U_{\infty}} = g(\xi)$ of this boundary layer developed between the plate leading-edge and far downstream as a function of $\xi = \frac{y}{\delta^*}$. For this purpose, we give the velocity profile $F_{\rm B}(\xi)$ of Blasius type laminar boundary layer formed over a flat plate without streamwise pressure gradient and without wall suction as a limiting form towards the plate leading-edge. We also give the velocity profile $F_{\rm f}(\xi)$ considered in Question 1 as a limiting form towards far downstream of the plate. By interpolating these two velocity profiles, we can obtain Equation (3) as a general form of velocity profile.

$$\frac{U}{U_{\infty}} = g(\xi) = (1 - K)F_{\rm B}(\xi) + KF_{\rm f}(\xi) \quad , \tag{3}$$

where K is a parameter and is a function of x.

When we denote $(g')_0$ and $(g'')_0$ as the values of $\frac{dg}{d\xi}$ and $\frac{d^2g}{d\xi^2}$ at $\xi = 0$, obtain the relationship between $(g')_0$ and $(g'')_0$ using Equation (2).

Question 3 We assume K is independent of ξ and, differentiation of Equation (3) leads to the following equations which hold at the plate surface, where $\binom{1}{2} = \frac{d}{d\xi} \binom{1}{\xi}$.

$$(g')_{0} = (1 - K)(F_{\rm B}')_{0} + K(F_{\rm f}')_{0}$$
⁽⁴⁾

$$(g'')_0 = (1 - K)(F_B'')_0 + K(F_f'')_0$$
(5)

1. Obtain $(F_{f}')_{0}$ and $(F_{f}'')_{0}$.

2. Show
$$\frac{V_0 \delta^*}{v} = \frac{K}{a + (1 - a)K}$$
 using $(F_{\rm B}'')_0 = 0$, where $a \equiv (F_{\rm B}')_0$.

Solid Mechanics (Morning)

In this problem, we consider a spherical symmetry problem of small deformation. Neglecting body forces, the equilibrium equation can be expressed as

and the strain-displacement relations are

$$\varepsilon_r = \frac{du}{dr}$$
, $\varepsilon_\theta = \frac{u}{r}$,(2)

where r is the distance from the center, σ_r and σ_{θ} are the stresses in the radial and the circumferential directions, respectively, ε_r and ε_{θ} are the strains in the radial and the circumferential directions, respectively and u is the displacement in the radial direction.

Question 1 Obtain the compatibility equation which ε_r and ε_{θ} must satisfy.

Question 2 In case of elastic body where Hooke's law holds, it is known that the general solution of the stresses can be expressed by the following equations,

$$\sigma_r = A + \frac{2B}{r^3} , \qquad \sigma_\theta = A - \frac{B}{r^3} , \qquad \dots \dots (3)$$

where A and B are arbitrary constants. It is obvious that these equations satisfy the equilibrium equation (1). Now, denoting Young's modulus and Poisson's ratio by E and ν , respectively, write down Hooke's law for the spherical symmetry problem. Further, show that the equation (3) satisfies the compatibility equation obtained in Question 1.

In the followings, consider an object with a spherical void of radius a. This object consists of an elasticperfectly plastic material which follows von Mises yield criterion with Young's modulus E and Poisson's ratio v in elastic deformation and the yield stress Y in uniaxial tension. Let a be sufficiently small compared with the size of the object. Here, "an elastic-perfectly plastic material which follows von Mises yield criterion" means that it yields when equivalent stress σ_{eq} becomes Y, and that $\sigma_{eq} = Y$ (constant value) in the plastically deformed region. Further, the equivalent stress σ_{eq} is defined by the following equation with three principal stresses σ_1 , σ_2 and σ_3 .

$$\sigma_{\rm eq} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \quad . \tag{4}$$

Question 3 Obtain the stress distribution during elastic deformation when internal pressure p acts in the void. When increasing p gradually, find the value of p at the onset of yielding (let the value be $p_{\rm Y}$).

Question 4 Letting r = c be the elastic-plastic boundary when $p > p_Y$, the region $a \le r \le c$ becomes the plastically deformed region and the region $r \ge c$ becomes the elastically deformed region. Find the value of σ_r at r = c.

Question 5 When p = Y in Question 4, obtain c and the stress distributions in the regions $a \le r \le c$ and $r \ge c$.

Question 6 Find the residual stress distribution when p becomes zero from the condition p = Y. Here, you may ignore Bauschinger effect.

Question 7 If this object is an elastic-perfectly plastic material following Tresca yield criterion, describe briefly the results of Questions 3 to 6 with the reason.

Aerospace System (Morning)

Let us consider a deep space probe departing from a low Earth orbit (circular orbit) to escape from Earth using a thruster with low thrust, such as electric propulsion. When considering a mission to gradually increase the orbital radius from the low Earth orbit by firing the thruster continuously and, after multiple orbits, to escape from the Earth's gravity sphere (let this orbital sequence be called "spiral ascent"), answer the following questions. You can only consider the in-plane orbital motion.

Question 1 When the orbital radius of the low earth orbit (circular orbit) is r and the Earth's gravitational constant is μ , find the orbital velocity v and the orbital period T of the circular orbit. Also show that the mechanical energy E of the orbit is expressed as follows.

$$E = -\frac{\mu}{2r}.$$

Question 2 The initial orbit is the low Earth orbit in question 1, and the low-thrust thruster is fired continuously with a constant acceleration a. The direction of the thrust is always controlled to be tangential to the orbit. The orbit change per orbital period is small (as seen in Figure 1) and follows the following equations, which are linearized around a circular orbit.

$$\frac{d^2x(t)}{dt^2} - 2\omega \frac{dy(t)}{dt} - 3\omega^2 x(t) = 0$$
$$\frac{d^2y(t)}{dt^2} + 2\omega \frac{dx(t)}{dt} = a$$

Here x(t) is the radial position, y(t) is the tangential position, and ω is the angular velocity of the reference orbit (circular orbit), respectively. Assuming that the orbit of the spacecraft starting from the reference orbit at time t = 0 (i.e., $\frac{dx(0)}{dt} = \frac{dy(0)}{dt} = x(0) = y(0) = 0$), find the change in altitude after one orbital period $\delta r(T) = x(T) - x(0)$. Also prove that the orbit after one orbital period becomes circular.

Question 3 Let $\delta v = aT$ denote the cumulative acceleration per orbital period. Using the relationship between δr and δv obtained from question 2, derive the total velocity change (ΔV) required for a spiral ascent from a circular orbit of radius r to a circular orbit of radius R (where R > r).

Question 4 Let us compare an electric propulsion spacecraft which departs from a low Earth orbit to escape from the Earth's gravity sphere $(R \to \infty)$ by spiral ascent with a low-thrust thruster and a conventional rocket which escapes from the Earth's gravity sphere by a single impulsive ΔV from the low Earth orbit with a high thrust rocket engine. Find the ratio of the ΔV required by the electric propulsion spacecraft to the ΔV required by the conventional rocket to escape from the Earth's gravity sphere.



Figure 1

Aerospace Propulsion (Morning)

As shown in Figure 1, an axisymmetric rigid object with a radius of a and a mass of m is kept stationary on a horizontal plane, and an impulse P is applied horizontally at time t = 0 to a point above the center O by h. Here, the point of impact and the center of gravity of the object are in the same vertical plane, and the density of the object is uniform.

Question 1. For each of the following cases, find h for this axisymmetric object to roll without slipping.

- 1. When the axisymmetric object is a disk.
- 2. When the axisymmetric object is a sphere.
- 3. When the axisymmetric object is a thin spherical shell.

In the following questions, let us consider arbitrary h ($0 \le h < a$). Here, the coefficient of dynamic friction between the axisymmetric object and the horizontal plane is μ , the gravitational acceleration is g, and the moment of inertia around the axis of rotation is I.

Question 2. Find the moving velocity u(t) of the center of gravity of the axisymmetric object and the rotational angular velocity $\omega(t)$ after the impact as a function of time.

Question 3. Find the condition of the impulse P for the axisymmetric object to climb the step of height H ($0 \le H < a$) shown in Figure 1. Here, the axisymmetric object starts rolling without slipping by the time it reaches the step, there is no friction between the axisymmetric object and the horizontal plane after that, and the axisymmetric object does not slip and does not bounce at the corner of the step.

